SIMPLE STRUCTURE
IN FACTOR ANALYSIS

A TERM PAPER

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Factor Analysis

by

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The Problem

There are quite a few ways of solving a factor problem. In each case, an investigator starts with a fundamental hypothesis about the nature of the phenomena to be measured. Quite likely, he is not trying to test the hypothesis that the ~~theoretical~~ factor structure is true. Rather he assumes the structure to be true and seeks merely to discover what the factors are. Presumably, if the method should fail, he would be "testing" the hypothesis of their existence indirectly.

The evidence for the foregoing assertions is very simple, i.e. each of the structures has a man's name connected with it as an advocate. Some generous souls are willing to use more than one method. This trend seems to be of more recent origin. The notion that each factor solution may have its place is not at all universal.

Much of the controversy seems to stem from basic philosophical differences. The metaphysical arguments center largely in the nature of reality. Does "G" with specific factors present an accurate picture of reality? Or, is reality better represented by a plurality of factors? The epistemological arguments seem to center around the nature of evidence and the regard one has for it. Does the exclusively mathematical solution give more reliable evidence than one that involves the subjective influence of the investigator? Is the invariant solution for factors to be prefered?

The focus of this paper is on the place of simple structure in factor analysis.

Some Alternative Models

Cattell (1952) compared the following seven final factor structures.

Spearman presented the idea that each test would contain one general factor and a specific factor. All the loadings are positive unless variables are reflected. Holzinger's bifactor model was shown to be like Spearman's with the exception that overlapping group factors were permitted. In this solution, the sign possibilities are the same as with Spearman. Holzinger's unifactor model consisted of nonoverlapping group factors with or without specific factors. Any signs were possible for the loadings. Cattell presented a fourth model which contained one general factor, overlapping group factors and specific factors. Any signs were possible. Hotelling and Kelley were shown to call for a model in which there were as many general factors as tests, a principle factor model. Positive and negative signs could be distributed irregularly throughout the structure. Burt's model used a general factor with ~~specific~~ nonoverlapping group factors, each of which had both positive and negative loadings. The model also had specific factors. Thurstone's model rotated to simple structure had overlapping groups and specific factors. Any signs were possible. In one sense the first factor can be considered a general factor since each successive factor is selected in such a way as to account for the maximum of variance.

This is the centroid analysis, but not simple structure for this requires (generally) rotation of the centroid solution to

Each of the above has definite advantages and disadvantages. Cattell has suggested that these direct factor & rotational models differ from each other chiefly in six ways. 1. Is the whole matrix analyzed at once or is it divided into subgroups? 2. Are the factor loadings obtained by simple or be weighted summations? 3. How many factors are required to explain the tests? 4. Is it necessary to rotate? 5. Is the method easy of difficult to compute? 6. How complete and accurate must the factor loadings be?

What is Simple Structure

The basic rationale for simple structure is ~~given best~~ perhaps best stated by (Cattell, 1952, pp 145-6)

> "Since unitary influences in nature may undoubtedly operate in a great variety of ways, sometimes overlapping in their influences, sometimes discrete, sometimes affectingall variables, sometines affecting only a few, the only constellation plan that is acceptable is one that is flexible to reality and capable of permitting the emergence of whatever natural structure exists in the data. The only method which will meet this requirement is one using rotation and, indeed, the unrotated bifactor and bipolar patterns as well as the principle axes solution as first obtained are ruled out at once. . Systems which restrict themselves less flexibly to one particular configuration of factors must be rejected, despite any claim to greater mathematical tidiness of ease of computation, because they require constellations such as nonoverlapping influences, which are contrary to all we know about the interaction of psychological and social forces.

(The basic rationale was given by Thurstone) or, at least, he is generally so credited.

Cattell sees simple structure as double parsimony. First, many variables are represented by a few factors. Second, the factors are so distributed as to get the simplest explanation. yeah? Simple structure assumes an underlying simple order in nature. Simple structure is the configurational representation of simple order.

Thurstone sets forth five criteria for simple structure. There should be at least one zero in each row of the oblique factor matrix so that the complexity of each test is less than the number of factors. Each column should have r linearly independent tests, i.e. the number of zero loadings should be equal to, or greater than, r. (where r= the number of factors) Each pair of columns should have several tests with zero in one column and a loading in the other. Each pair of columns should have several tests

with zero in both columns if r is four or greater. For every pair of columns a small number of tests should have non-zero loadings in both.

Geometrical Representations

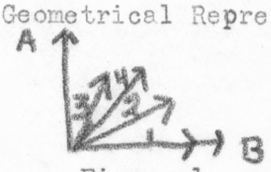

Figure 1

Where there are only two factors the structure may be represented in two dimensions. The factors A and B are the axes and represent unit vectors. The loadings of the four tests are represented by the projections. The correlation between two variables is given by the product of the two test vectors, i.e. $h_j h_k \cos \theta_{jk}$. Since the axes represent only one of the infinite ways of looking at the structure, and the structure is fixed, neither the correlation between variables not the communalities are changed when the axes are rotated. The loadings will change. A rigid right angle rotation of axes is called orthogonal rotation. The factors are uncorrelated in that setting. When the axes are permitted to be less than ninety degrees apart the rotation is oblique and the factors are correlated.

It is also possible to represent a three factor structure. This must be done in three dimensions with a sphere. A right spherical triangle represents the three planes with its sides. The angles represent the factors. Since these factors are unit vectors they reach the surface of the sphere from the origin at the center of the sphere. Since the test vectors are usually of less than unit length they must be brought up to unit length, normalized.* Each loading in the structure multiplied by $1/h_j$ will give a new structure in which each vector is of unit length which can be plotted

relative to the right spherical triangle. (for orthogonal axes) When the number of factors exceeds three the hyperspace must be viewed two or three dimensions at a time.

Properties of Simple Structure

Thurstone (1961) saw the following properties of simple structure. Simple structure is not unique. There is room for subjective interpretation in it. However, Thurstone argues that one can get uniqueness in a given matrix if the stated criteria are used. Simple structure implies that test vectors lie in cordinate planes or hyperplanes but not that factor loadings are all positive. Simple structure is invariant, i.e. loadings of a set of variables form into the same factors regardless of what additional variables are included. In other words, the effect of the addition of other variables is the overdetermine the plane. This holds true for the same population only. A confuguration has $\frac{1}{2}$ r(r-1) plots that must conform to the stated criteria for simple structure. A zero loading does not imply the absense of a factor in a variable, it means only that the factor does not contribute to the variance of the variable. Finally, although in the early factor analytic work with abilities positive manifold was found to exist, simple structure does not require it to be so.

Checking the Solution After Rotation to Simple Structure

The completion of rotation does not mean completion of the solution. Thurstone says that the following questions must be asked. Is the solution plausable for each hyperplane? Is the solution plausable theoretically? Do the factors predict well? How convincing is the simple structure? The

latter question can be answered by examination of the plots of vectors. Most of these checks involve a high degree of subjectivity.

Rotation

Eysenck (1950) reports that there are at least seven general criteria for rotation: (1) rotation to agree with clinical and general psychological findings; (2) rotation to agree with factors from past factor analyses; (3) rotation to put axes through the center of clusters; (4) the principle of orthogonal additions, rotations to agree with successively established factors; (5) rotations to produce loading profiles congruent with general psychological expectation; (6) rotation to simple structure for a given matrix; and (7) the principle of proportional profiles, or " simultaneous simple structure." Eysenck reduces these to two, rotation to some outside criterion or rotation based on the matrix.

Issues and Problems

Perhaps it is an understatement to say that simple structure is not universally acknowledged as the ideal factor solution. The following is a list of some of the problems and issues.

1. Some authorities complain that the investigator can make simple structure simply by choosing the tests. This would be dishonest if it were not reported but it would be of value to know that such a skill existed.

2. Simple structure yields only the factors put in the battery and not what nature says. This objection seems of little consequence since the argument about what nature "really is" has no apparent means of proof. If the factors in the selected battery explain that battery the results will be useful.

3. Thurstone presented his box problem in an attempt to verify the legitimacy of simple structure. He found that simple structure factoring was sensitive to the location of the measurement factors in randomly collected boxes.

4. Kaiser (1958) criticized rotation for meaning as being subjective and therefore exclusive of a unique solution. The reasoning is seen as a posteriori. He calls for an alternative. "In contrast, an analytic criterion for rotation would allow factor analysis to become a straightforward methodology stripped of its subjectivity and a proper tool for scientific inquiry. "Kaiser says the ultimate criterion of a rotational proceedure is not simple structure but factorial invariance.

5. Does simple structure describe nature or do the other structures do a better job of description. All of the structures are probably needed for the best description of "nature." All of the solutions become inadequate when someone advocates them as the only solution. In effect, this is defining in advance the answer to a problem.

6. Thompson (1962) presents the following views. Simple structures uniqueness will be supported with the application of computers to the criteria, even for correlated factors. To five above, Thompson answers that the problem of ~~true~~ best reflection of reality will always be with us even though we have unique solutions. These are judgmental, the problem of integrating results of studies covering unrelated objects of a problem area.

7. Reyburn and Raath (1949) point up a basic problem in the rationale of simple structure. They find that oblique simple structure complicates matters by requiring one or more additional factors to account for their own intercorrelations. Pure simple structure must be orthogonal; for

No, but this is a common misunderstanding. What is meant is that it is not necessary to imply that new factors must be introduced to account for correlations between factors. e.g., in theory one could have $m \leq n$ linearly independent components (factors) which, depending on theory, could be correlated or not correlated.

only so can all the factors conform to the requirements which simple structure implies. But, this is unpsychological since behavior is likely to be the result of related factors.

8. Schmid argues for oblique simple structure. He feels that the parsimonious orthogonality is unnatural. So the principle of related factors is necessary. However, the present method of rotation is inadequate.

9. A modification of the criteria for simple structure may be necessary in special cases. Cattell (1946) feels that the proportion of zero loadings will differ with the kind of thing measured. For example the number of zero loadongs on personality variables is likely to be less than with abilities since personality traits have such "wide influence." He suggests that ¼ to 1/3 of the loadings in a randomly selected, widely chosen set of personality measures might be expected to have zero loadings.

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